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# Stolarsky-3 Mean Labeling of Some Special Graphs

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**Abstract**

Let G = (V, E) be a graph with p vertices and q edges . G is said to be Stolarsky-3 Mean graph if each vertex x∈V is assigned distinct labels f(x) from 1,2,….,q+1 and each edge e=uv is assigned the distinct labelsf(e=uv) = $\left\lceil\sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}}\right\rceil$ (or) $\left\lfloor\sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}}\right\rfloor$ then the resulting edge labels are distinct. In this case f is called a Stolarsky-3 Mean labeling of G and G is called a Stolarsky-3 Mean graph. In this paper we investigate the Stolarsky-3 Mean labeling of some special graphs.

**Keywords:** Graph Labeling, Mean Labeling, Stolarsky-3 Mean Labeling, Slanting Ladder, Triangular Ladder, H-graph, Twig graph, Middle graph, Total graph.

## 1. INTRODUCTION

The graphs G = (V,E) considered in this paper are finite, undirected and without loops or multiple edges. We follow Gallian**[1]** for all detailed survey of graph labeling and we refer Harary**[2]** for all other standard terminologies and notations. The concept of **"Mean Labeling of graphs"** has been introduced S. Somasundaram, R.Ponraj and S.S.Sandhya in 2004**[3]** and S.Somasundaram and S.S. Sandhya introduced the concept of **"Harmonic Mean Labeling of graphs" in[4]. "Stolarsky-3 Mean Labeling of graphs"** was introduced by S.S. Sandhya, E.Ebin Raja Merely and S.Kavitha [7].

The following definitions are necessary for the present study.

**Definition 1.1:** A graph G with p vertices and q edges is said to be Stolarsky-3 Mean graph if each vertex x∈V is assigned distinct labels f(x) from 1,2,….,q+1 and each edge e=uv is assigned the distinct labels f(e=uv) = $\left\lceil\sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}}\right\rceil$ (or) $\left\lfloor\sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}}\right\rfloor$ then the resulting edge labels are distinct. In this case f is called a Stolarsky-3 Mean labeling of G.

**Definition 1.2:** The Slanting ladder $SL_n$ is a graph obtained from two points $u_1, u_2, \dots, u_n$ & $v_1, v_2, \dots, v_n$ by joining each $u_i$ with $v_{i+1}$ $1 \le i \le n-1$.

**Definition 1.3:** A Triangular ladder is a graph obtained from $L_n$ by adding the edges $u_i v_{i+1}$, $1 \le i \le n-1$, where $u_i$ and $v_i$ $1 \le i \le n$ are the vertices of $L_n$ such that $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_n$ are two paths of length n in the graph $L_n$.

**Definition 1.4:** The H-graph of a path $P_n$ is the graph obtained from two copies of $P_n$ with vertices $v_1, v_2, v_3, \dots, v_n$ & $u_1, u_2, \dots, u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ & $u_{\frac{n+1}{2}}$ if n is odd and the vertices $v_{\frac{n}{2}+1}$ & $u_{\frac{n}{2}}$ if n is even.

**Definition 1.5: :** The Middle graph M(G) of a graph G is the graph whose vertex set is V(G)∪E(G) and in which two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident on it.

**Definition 1.6:** The Total graph T(G) of graph G is the graph whose vertex set is V(G)∪E(G) and two vertices are adjacent whenever they are either adjacent or incident in G.

**Definition 1.7:** A graph G (V,E) obtained from a path by attaching exactly two pendant edges to each interval vertices of the path is called a Twig graph.

## 2. MAIN RESULTS

**Theorem 2.1**: Slanting Ladder $SL_n$ is Stolarsky-3 Mean graph.

**Proof:** Let G be the slanting ladder graph with the vertices $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_n$.

Define a function **f** : V(G) →{ 1,2,…., q+1} by

$\mathbf{f}(u_i)$ = 3i , $1 \le i \le n-1$.

$\mathbf{f}(u_n)$ = 3n -2 .

$\mathbf{f}(v_1)$ = 1 .

$\mathbf{f}(v_i)$ = 3i -4 , $2 \le i \le n$.

Then the edges are labeled with

$\mathbf{f}(u_i u_{i+1})$ =3i+1, $1 \le i \le n-1$.

$\mathbf{f}(u_i v_{i+1})$ =3 i-1, $1 \le i \le n-1$.

$\mathbf{f}(v_1 v_2)$ =1.

$\mathbf{f}(v_i v_{i+1})$ =3(i -1), $2 \le i \le n-2$.

Then the edge labels are distinct.

Hence $SL_n$ is Stolarsky-3 Mean graph.

**Example 2.2:** The Stolarsky-3 Mean labeling of $SL_6$ is given below.

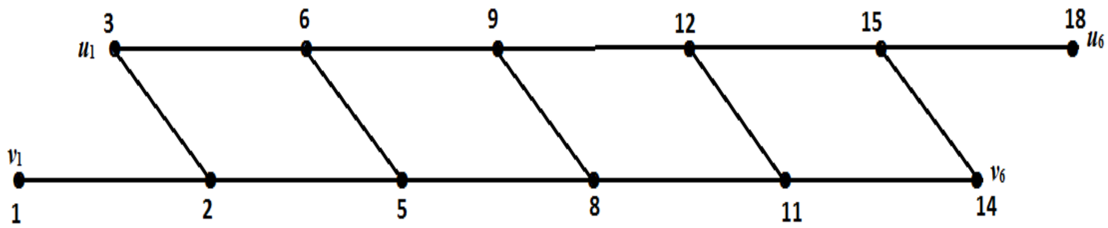



**Figure:1**

**Theorem 2.3:** Triangular Ladder $TL_n$ is Stolarsky-3 Mean graph.

**Proof:** Let $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ be two paths of length n.

Join $u_i$ and $v_i$, $1 \le i \le n$ , and join $u_i$ and $v_{i+1}$, $1 \le i \le n-1$. The resulting graph is $TL_n$.

Define a function **f** : $V(TL_n) \to \{1,2,\ldots, q+1\}$ by

**f**$(u_i)$ = 4i -2 , $1 \le i \le n$.

**f**$(v_1)$ = 1.

**f**$(v_i)$ = 4(i-1) , $2 \le i \le n$.

Then the edges are labeled with

**f**$(u_i u_{i+1})$ =4i , $1 \le i \le n-1$.

**f**$(u_i v_i)$ =4 i -3, $1 \le i \le n$.

**f**$(v_i v_{i+1})$ =4i-2, $1 \le i \le n-1$.

**f**$(u_i v_{i+1})$ =4i-1, $1 \le i \le n-1$.

Then the edge labels are distinct.

Hence $TL_n$ is Stolarsky-3 Mean graph.

**Example 2.4:** The Stolarsky-3 Mean labeling of $TL_6$ is given below.

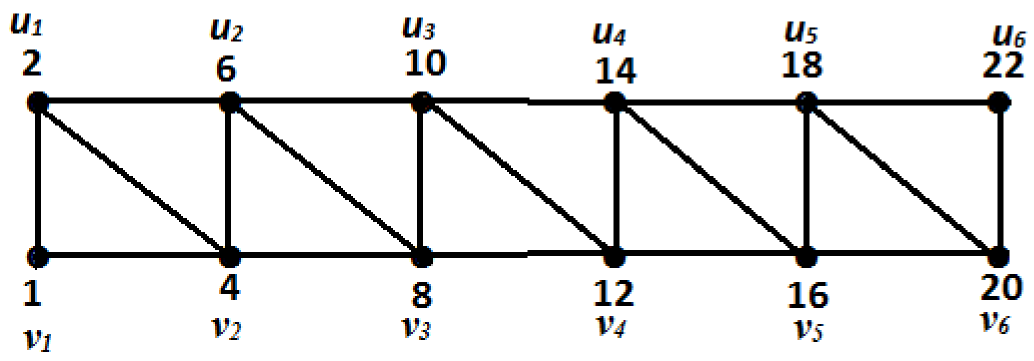



**Figure:2**

**Theorem 2.5:** H graph is Stolarsky-3 Mean graph for all n if n is even and $n \le 11$ if n is odd.

**Proof:** Let G be the graph with the vertices $v_1, v_2, \ldots, v_n$ & $u_1, u_2, \ldots, u_n$.

Define a function **f** : $V(G) \to \{1,2,\ldots, q+1\}$ by

$\mathbf{f}(v_i)$ = i, 1≤ $i \leq n$.

$\mathbf{f}(u_i)$ = n +i , 1≤ $i \leq n$.

Then the edges are labeled as

$\mathbf{f}(v_i v_{i+1})$ = i , 1≤ $i \leq n-1$.

$\mathbf{f}(u_i u_{i+1})$ =n +i, 1≤ $i \leq n-1$.

$\mathbf{f}( v_{\frac{n+1}{2}} u_{\frac{n+1}{2}})$ = n if n is odd.

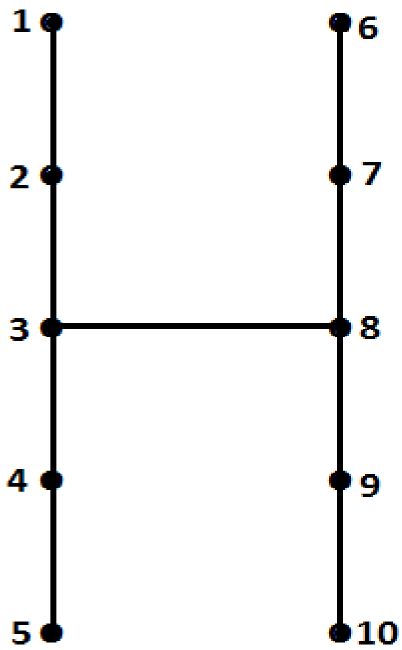
$\mathbf{f}(v_{\frac{n}{2}+1} u_{\frac{n}{2}})$ = n if n is even.

Then we get distinct edge labels.

Hence **f** is Stolarsky-3 Mean labeling.

**Example 2.6:** The labeling pattern of H graph is given below.

**When n=5**



**When n=6**

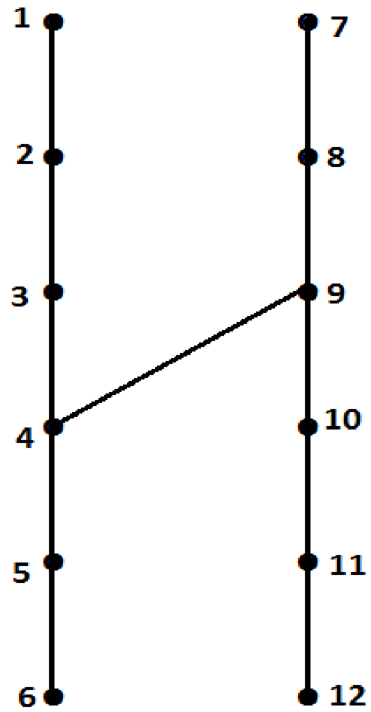



**Figure: 3**

**Theorem 2.7:** Twig graph $T_m$ is Stolarsky-3 Mean graph.

**Proof:** Let $G$ be the twig graph.

Let $u_1, u_2, \dots, u_n$ be the vertices of the path $P_n$ and $v_1, v_2, \dots, v_{n-2}$& $w_1, w_2, \dots, w_{n-2}$ be two pendant vertices attached to $u_i$.

Define a function **f** : V(G) →{ 1,2,…., q+1} by

$\mathbf{f}(u_1)$ = 1.

$\mathbf{f}(u_i)$ =3i-4, $2 \leq i \leq n$.

$\mathbf{f}(v_i)$ = 3i, $1 \leq i \leq n-2$.

$\mathbf{f}(w_i)$ = 3i +1 , $1 \leq i \leq n-2$.

Then the edges are labeled with

$\mathbf{f}(u_i u_{i+1})$ =3i -2, $1 \leq i \leq n-1$.

$\mathbf{f}(v_i u_i)$ =3 i -1, $1 \leq i \leq n-2$.

$\mathbf{f}(w_i u_i)$ =3 i, $1 \leq i \leq n-2$.

Then the edge labels are distinct.

Hence **f** is Stolarsky-3 Mean labeling.

**Example 2.8:** The Stolarsky-3 Mean labeling of Twig graph $T_3$ is given below.

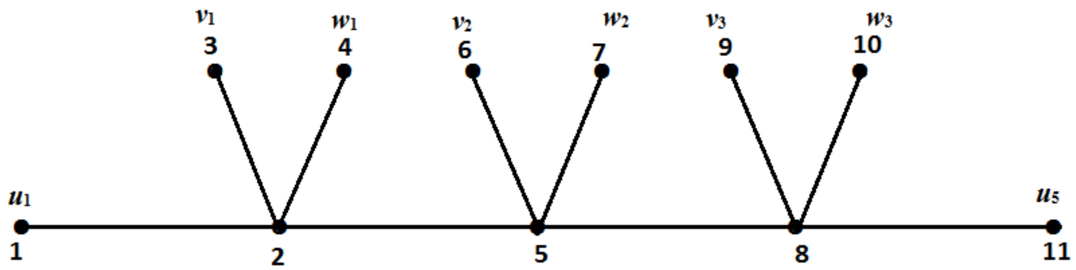



**Figure 4**

**Theorem 2.9:** Middle graph M( $P_n$) is Stolarsky-3 Mean graph.

**Proof:** Let $u_1, u_2, \dots, u_n$ & $v_1, v_2, \dots, v_{n-1}$ be the vertices of the middle graph G=M($P_n$).

By definition of middle graph V(M($P_n$)) = V($P_n$) ∪E($P_n$) and whose edge set is

$$E(M(P_n))=\begin{cases} u_i v_i \text{ , } 1 \le i \le n-1 \\ u_i v_{i-1} \text{ , } 2 \le i \le n \\ v_i v_{i+!} \text{ , } 1 \le i \le n-2 \end{cases}$$

Here $|V(G)|$ = 2n-1 and $|E(G)|$ = 3n-4.

We define f: V(G)→ {1,2,3,….,q+1} by

$\mathbf{f}(u_1)$ = 1.

$\mathbf{f}(u_i)$ =3i, 2≤ $i \le n$.

$\mathbf{f}(v_i)$ = 3i-1, 1≤ $i \le n-1$.

Then the edges are labeled with

$\mathbf{f}(u_i v_i)$ =3i -2, 1≤ $i \le n-1$.

$\mathbf{f}(u_i v_{\boldsymbol{i-1}})$ =3 i -1, 2≤ $i \le n-1$.

$\mathbf{f}(v_i v_{\boldsymbol{i+1}})$ =3 i, 1≤ $i \le n-2$.

Then the edge labels are distinct.

Hence Middle graph M( $P_n$) is stolarsky-3 Mean graph.

**Example 2.10:** The Stolarsky-3 Mean labeling of M($P_6$) is given below.

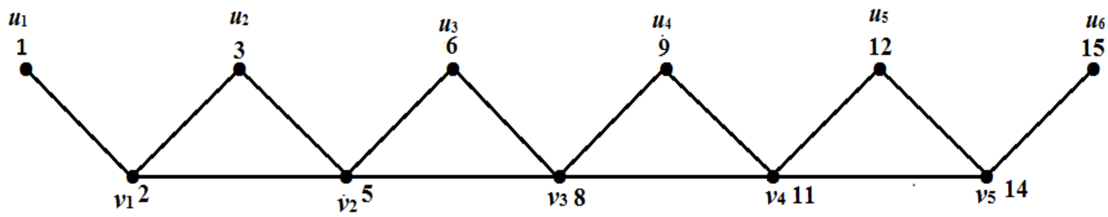



**Figure: 5**

**Theorem 2.11:** Total graph T( $P_n$) is Stolarsky-3 Mean graph.

**Proof:** Let $u_1, u_2, \dots, u_n$ & $v_1, v_2, \dots, v_{n-1}$ be the vertices of the Total graph T($P_n$).

By definition of Total graph V (T ($P_n$)) = V ($P_n$) ∪ E ($P_n$) and

$$E(T(P_n)) = \begin{cases} u_iu_{i+1}, 1 \le i \le n-1. \\ u_iv_i, 1 \le i \le n-1. \\ u_iv_{i-1}, 2 \le i \le n. \\ v_iv_{i+1}, 1 \le i \le n-2. \end{cases}$$

Here $|V(G)|$=2n-1 and $|E(G|$ = 4n-5.

Define f : V(G) → {1,2,……,q+1}as follows.

**f**$(u_1)$ = 2.

**f**$(u_i)$ =4i-1, 2≤ $i \le n$.

**f**$(v_i)$ = 4i-3, 1≤ $i \le n-1$.

Then the edges are labeled with

**f**$(u_iu_{i+1})$ =4i -2, 1≤ $i \le n-1$.

**f**$(u_iv_i)$ =4i -3, 1≤ $i \le n-1$.

**f**$(u_iv_{i-1})$ =4i -2, $2 \le i \le n$.

**f**$(v_iv_{i+1})$ =4i , 1≤ $i \le n-2$.

Then the edge labels are distinct.

Hence T ($P_n$) is Stolarsky-3 Mean graph.

**Example 2.12:** The Stolarsky-3 Mean labeling of $T(P_6)$ is given below.

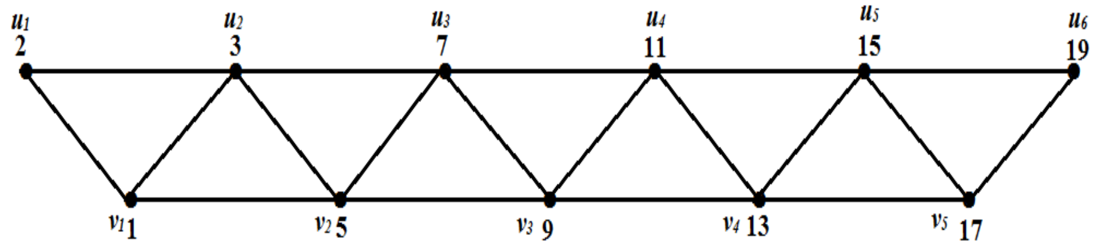



**Figure: 6**

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